

## On Approach for Analysis of Random Economic Processes

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**Abstract:** In this paper we introduce a model for prognosis of random economical processes. The approach gives a possibility to make prognosis of the considered processes during changing of economical situation. Also we introduce an analytical approach for analysis of the introduced model.

**Keywords:** random economical processes; approach for prognosis.

### Introduction

Economic systems are subject to the influence of a large number of uncontrollable external factors (weather conditions, foreign policy, social factors, deliberate distortion and concealment of information for the purpose of economic sabotage, etc.) [1-5]. In this situation, the system parameters can take random values. This paper considers a model for prognosis of processes in economic systems with random values of parameters. We also introduce an analytical approach for analysis of the model.

### Method of Solution

Let us consider a random process  $h(t)$ . Let us fix moments of time  $t_1, t_2, \dots, t_n$ . Probability of obtaining of value  $h(t_n)$  in the following interval  $h(t_n) \in [x_n, x_n + dx_n]$  if moment  $t_1$  random process assumes value  $x_1$ , if moment  $t_2$  random process assumes value  $x_2, \dots$ , if moment  $t_{n-1}$  random process assumes value  $x_{n-1}$  is equal to the product of the conditional probability density and the length of the interval:  $P = W(x_1, t_1; x_2, t_2; \dots; x_{n-1}, t_{n-1} | x_n, t_n) dx_n$ . In the common case the considered probability depends on all values  $x_n, t_n$ . We consider a model with higher exactness. At the same time analysis of the model is harder. In this situation it is attracted an interests model with compromise:

the model gives a possibility to simplify prognosis of processes with maximal saving of adequateness of description. As one of most simple models it could be consider Markov process: we consider only one previous value  $x_{n-1}$  during analysis of the considered process. Probability density of the Markov process describes by the Focker -Planck equation [6]

$$\frac{\partial W(x,t)}{\partial t} = \frac{\partial^2 [D(x,t) W(x,t)]}{\partial x^2} - \frac{\partial [K(x,t) W(x,t)]}{\partial x}, \quad (1)$$

where  $D(x,t)$  is the diffusion coefficient,  $K(x,t)$  is the drift coefficient. To illustrate the introduced method of solution of the equation (1) we consider the following boundary and initial conditions

$$W(0,t)=0; W(L,t)=0; W(x,0)=f(x). \quad (2)$$

At variable values of coefficients of diffusion and drift one can obtain exact solution of the equation (1) only for few dependences of the considered coefficients on variables  $x$  and  $t$ . In this situation we consider an approach to obtain approximate solution of the considered equations. To describe the approach we transform coefficients  $D(x,t)$  and  $K(x,t)$  in the following forms

$$D(x,t)=D_0[1+\varepsilon g(x,t)], K(x,t)=K_0[1+\xi h(x,t)], \quad (3)$$

where  $|g(x,t)| \leq 1$ ,  $|h(x,t)| \leq 1$ ,  $0 \leq \varepsilon < 1$ ,  $0 \leq \xi < 1$ ,  $D_0$  and  $K_0$  are the average values of the considered coefficients. Next we will determine solution of the equation (1) as the following series

$$W(x,t) = \sum_{k=0}^{\infty} \varepsilon^k \sum_{l=0}^{\infty} \xi^l W_{kl}(x,t). \quad (4)$$

Substitution of the above series and relations (3) into the equation (1) leads to the following result

$$\begin{aligned} \sum_{k=0}^{\infty} \varepsilon^k \sum_{l=0}^{\infty} \xi^l \frac{\partial W_{kl}(x,t)}{\partial t} = D_0 \sum_{k=0}^{\infty} \varepsilon^k \sum_{l=0}^{\infty} \xi^l \frac{\partial^2 \{ [1 + \varepsilon \cdot g(x,t)] W_{kl}(x,t) \}}{\partial x^2} - \\ - K_0 \sum_{k=0}^{\infty} \varepsilon^k \sum_{l=0}^{\infty} \xi^l \frac{\partial \{ [1 + \xi \cdot h(x,t)] W_{kl}(x,t) \}}{\partial x}. \end{aligned} \quad (5)$$

Grouping the coefficients for the same powers of the parameters  $\varepsilon$  and  $\xi$  makes it possible to obtain equations for the functions  $W_{kl}(x,t)$

$$\begin{aligned} \frac{\partial W_{kl}(x,t)}{\partial t} = D_0 \frac{\partial^2 W_{kl}(x,t)}{\partial x^2} + D_0 \frac{\partial^2 [g(x,t) \cdot W_{k-l}(x,t)]}{\partial x^2} - \\ - K_0 \frac{\partial W_{kl}(x,t)}{\partial x} - K_0 \frac{\partial [h(x,t) \cdot W_{k-l-1}(x,t)]}{\partial x}, \quad k \geq 0, l \geq 0. \end{aligned} \quad (6)$$

Boundary and initial conditions for the functions  $W_{kl}(x,t)$  could be written in the form

$$W_{kl}(0,t) = 0, W_{kl}(L,t) = 0, k \geq 0, l \geq 0; W_{00}(x,0) = f(x), W_{kl}(x,t) = 0, k \geq 1, l \geq 1. \quad (7)$$

Equations (6) with conditions (7) could be solved, for example, by the Fourier approach. The obtained solutions could be written as

$$\begin{aligned} W_{00}(x,t) &= \sum_{n=0}^{\infty} e^{-\frac{\pi^2 n^2 D_0^2 + K_0^2 L^2}{4 D_0 L^2} t} \sin\left(\frac{\pi n x}{L}\right) \int_0^L f(x) \sin\left(\frac{\pi n x}{L}\right) dx \\ W_{10}(x,t) &= \frac{1}{2} \sum_{m=0}^{\infty} e^{-\frac{\pi^2 m^2 D_0^2 + K_0^2 L^2}{4 D_0 L^2} t} \sin\left(\frac{\pi m x}{L}\right) \sum_{n=0}^{\infty} e^{-\frac{\pi^2 n^2 D_0^2 + K_0^2 L^2}{4 D_0 L^2} t} \int_0^L f(x) \sin\left(\frac{\pi n x}{L}\right) dx \times \\ &\times \int_0^L g(x,t) \left\{ \cos\left[\frac{\pi(m-n)x}{L}\right] - \cos\left[\frac{\pi(m+n)x}{L}\right] \right\} dx \\ W_{01}(x,t) &= -\frac{1}{2} \sum_{m=0}^{\infty} e^{-\frac{\pi^2 m^2 D_0^2 + K_0^2 L^2}{4 D_0 L^2} t} \sin\left(\frac{\pi m x}{L}\right) \sum_{n=0}^{\infty} e^{-\frac{\pi^2 n^2 D_0^2 + K_0^2 L^2}{4 D_0 L^2} t} \int_0^L f(x) \sin\left(\frac{\pi n x}{L}\right) dx \times \\ &\times \int_0^L h(x,t) \left\{ \cos\left[\frac{\pi(m-n)x}{L}\right] - \cos\left[\frac{\pi(m+n)x}{L}\right] \right\} dx \\ W_{11}(x,t) &= \frac{1}{2} \sum_{m=0}^{\infty} e^{-\frac{\pi^2 m^2 D_0^2 + K_0^2 L^2}{4 D_0 L^2} t} \sin\left(\frac{\pi m x}{L}\right) \sum_{n=0}^{\infty} e^{-\frac{\pi^2 n^2 D_0^2 + K_0^2 L^2}{4 D_0 L^2} t} \int_0^L g(x,t) \left\{ \cos\left[\frac{\pi(m-n)x}{L}\right] - \cos\left[\frac{\pi(m+n)x}{L}\right] \right\} dx \times \\ &\times \int_0^L f(x) \sin\left(\frac{\pi n x}{L}\right) dx - \frac{1}{2} \sum_{k=0}^{\infty} e^{-\frac{\pi^2 k^2 D_0^2 + K_0^2 L^2}{4 D_0 L^2} t} \sum_{l=0}^{\infty} e^{-\frac{\pi^2 l^2 D_0^2 + K_0^2 L^2}{4 D_0 L^2} t} \int_0^L h(x,t) \left\{ \cos\left[\frac{\pi(k-l)x}{L}\right] - \cos\left[\frac{\pi(k+l)x}{L}\right] \right\} dx \times \\ &\times \sin\left(\frac{\pi k x}{L}\right) \int_0^L f(x) \sin\left(\frac{\pi l x}{L}\right) dx \end{aligned}$$

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The features of this method are:

- (i) the ability to solve equation (1) for any values of the parameters  $e$  and  $x$  due to the impossibility of taking negative values by the

coefficients  $D(x,t)$  and  $K(x,t)$ : series (4) is convergent. At the same time, this series has an infinite number of terms. In this situation, series (4) cannot be used in full, which leads to the need to approximate the solution of equation (1) by the finite sum of series (4);

- (ii) using of the considered method of solution with discontinuous coefficients  $D(x,t)$  and  $K(x,t)$  makes it possible to obtain a solution to equation (1) without using the matching of solutions at discontinuity points [8,9]. The feature gives a possibility to reduce the volume of transformations [8,9].

## Conclusion

We introduce model for prognosis of random economical processes. Based on the approach we have a possibility to make prognosis of the above processes in the framework of changing of economical situation. Also we introduce an analytical approach for analysis of the introduced model.

## References

- [1] S. Nathaphan, P. Chunhachinda. Economics research international. **Vol. 2010**. Article ID 340181 (2010).
- [2] J. Prat. Mathematics and financial economics. **Vol. 9** (3). P. 169-193 (2015).
- [3] Ch.W. Wu, W.Zh. Hung. Economics research international. **Vol. 2010**. Article ID 678927 (2010).
- [4] St. Villeneuve, X. Warin. Mathematics and financial economics. **Vol. 8** (2). P. 193-227 (2014).
- [5] Ch. Mertzanis. Studies in economics and finance. **Vol. 34** (1). P. 82-104 (2017).
- [6] H. Risken. *The Fokker-Planck equation: methods of solution and applications* (Berlin: Springer-Verlag, 1984).
- [7] A.N. Tikhonov, A.A. Samarsky. *The equations of mathematical physics* (Moscow: Science, 1972).
- [8] E.L. Pankratov. Physical Review B. **Vol. 72** (7). P. 075201-075208 (2005).
- [9] E.L. Pankratov, E.A. Bulaeva. Reviews in theoretical science. **Vol. 1** (1). P. 58-82 (2013).